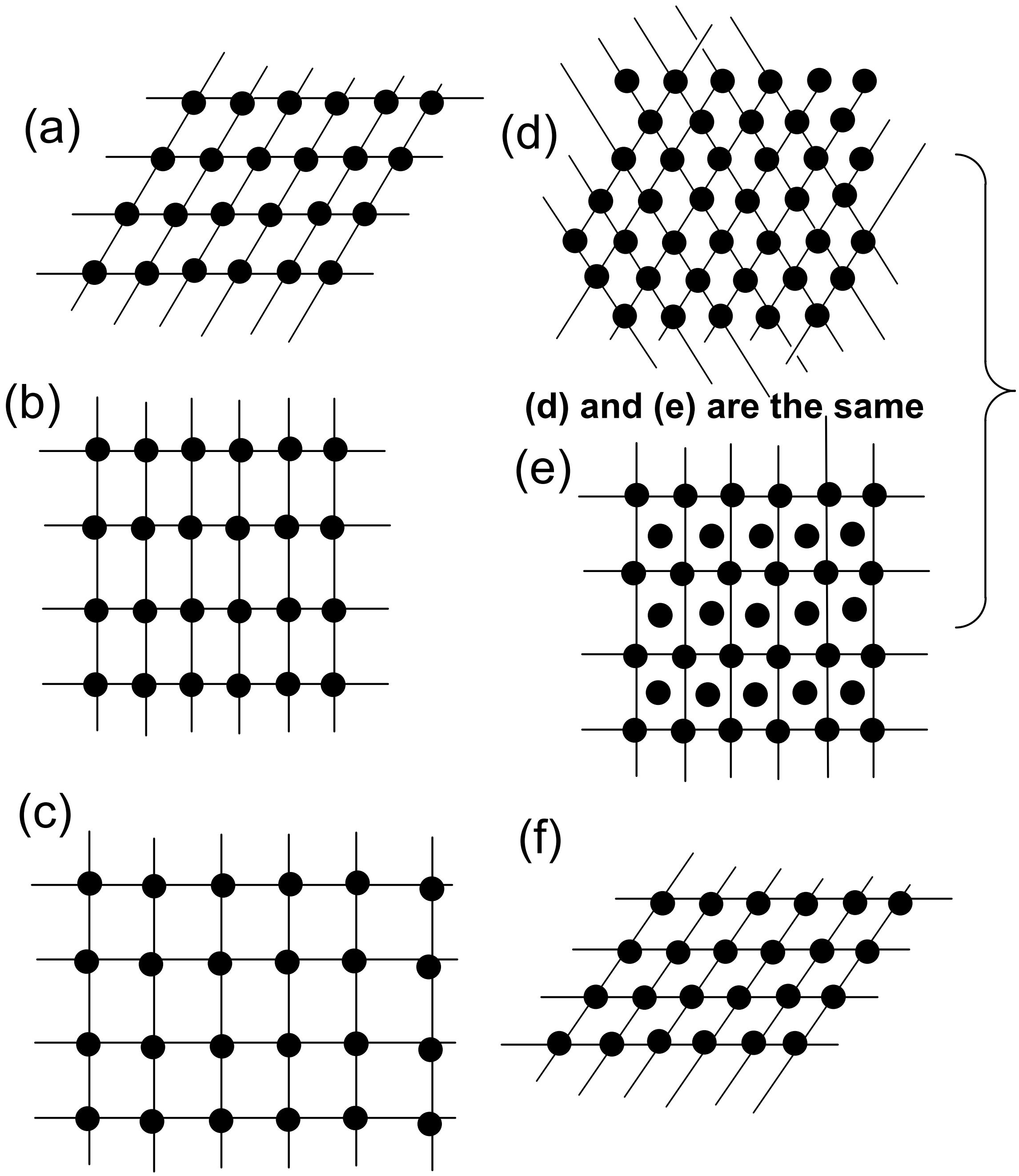
THE FIVE TYPES OF PLANAR 2-D **LATTICES**



(a) **OBLIQUE LATTICE** - NO RESTRICTIONS ON ANGLES BETWEEN THE UNIT CELL EDGES

(b) **RECTANGULAR LATTICE** - ANGLE BETWEEN UNIT CELL EDGES ARE 90 DEG.

(c) **SQUARE LATTICE** - ANGLE BETWEEN UNIT CELL EDGES ARE 90 DEG.AND a = b

(d), (e) **CENTERED LATTICE**

TWO WAYS TO DEPICT IT.

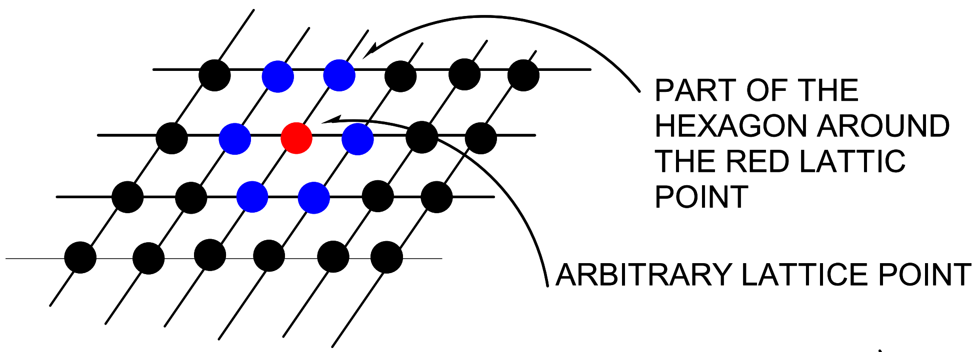
(d) a=b, BUT ANGLE IS NOT 90 DEG.

(e) a=b, WITH 90 DEG. ANGLE AND A LATTICE POINT IN THE CENTER

(PRIMITIVE AND CENTERED ARE DIFFERENT TYPES OF UNIT CELLS IN 2-D AND 3-D. PRIMITIVE HAS LATTICE POINTS ONLY AT THE CORNERS WHEREAS CENTERED HAS ONE IN THE CENTER AS WELL. THE POINT IN THE CENTER IS NOT DIFFERENT THAN THE ONES AT THE CORNERS - IT IS GENERATED BY CORNER POINTS.

(f) **HEXAGONAL SYMMETRY** - ANGLES BETWEEN UNIT CELL EDGES ARE 60 DEG. OR 120 DEG. (SIX-FOLD ROTATIONAL SYMMETRY)

IN THE HEXAGONAL 2-D LATTICE, ANY GIVEN LATTICE POINT HAS SIX POINTS AROUND IT IN THE SHAPE OF A HEXAGON, i.e., IT HAS A SIX-FOLD ROTATION AXIS PERPENDICULAR TO THE PLANE



(IN FACT, EACH OF THE FIVE, 2-D LATTICE HAVE ROTATIONAL AXES THAT ARE PERPENDICULAR TO THE PLANE)

SYMBOLS USED TO DENOTE SYMMETRY IN LATTICES:

n = ROTATIONAL AXIS OF ORDER "n"
m = MIRROR LINES OR PLANES
P = PRIMITVE LATTICE
C = CENTERED LATTICE

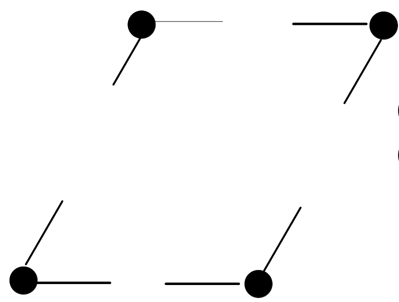
PICTORIAL REPRESENTATIONS:

ROTATION AXES PERPENDICULAR TO THE PLANE OF THE PAPER

A LINE DENOTES A MIRROR

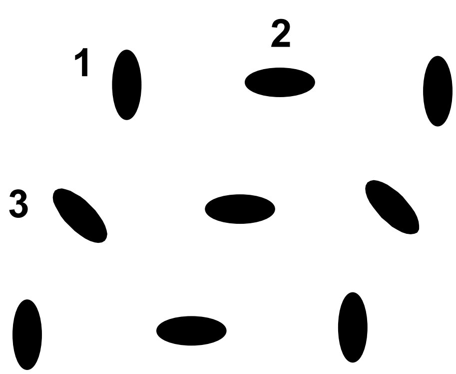
***ONE CANNOT HAVE 5-FOLD, 7-FOLD, 8-FOLD ETC., ROTATIONAL SYMMETRY IN A LATTICE**
WHY? PROVE IT

1. p2 - Oblique 2-D lattice



ONE UNIT CELL OF THE OBLIQUE LATTICE

CAN BE DRAWN AS

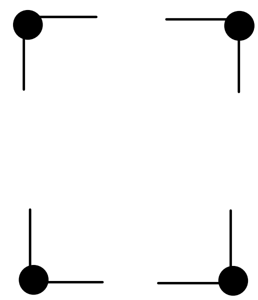


These symbols are for three different types of C_2 axes they are drawn in three different orientations so one can distinguish them from each another

SYMBOL is p2

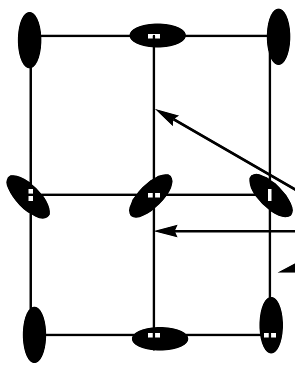
p means primitive and C_2 is the only symmetry element

2. pmm(2) - Rectangular 2-D lattice



ONE UNIT CELL OF THE RECTANGULAR LATTICE

CAN BE DRAWN AS:



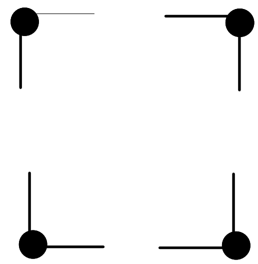
This collection of symbols denotes four different types of <u>C_2 axes</u> and the <u>mirrors</u> which are solid lines

SYMBOL is pmm(2)

p means primitive and two m symbols mean two types of mirror planes (the 2 in parentheses is redundant because the C_2 axes are generated by the mirrors)

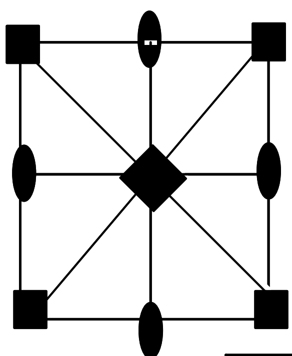
the minimum amount of symmetry to generate the rest is used

3. p4(mm2) - Square 2-D lattice



ONE UNIT CELL OF THE SQUARE LATTICE

CAN BE DRAWN AS:

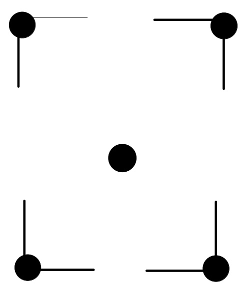


This collection of symbols denotes two different types of <u>C_4 axes, one type of C_2 axis</u> and <u>two types of mirrors</u> which are solid lines

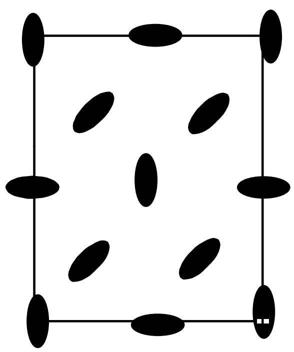
SYMBOL is p4(mm2)

p means primitive and the 4 means a C_4 axis (the m's and 2 in parentheses are redundant because they are generated by the C_4)

4. cmm(2) - Centered Rectangular 2-D lattice



ONE UNIT CELL OF THE CENTERED RECTANGULAR LATTICE

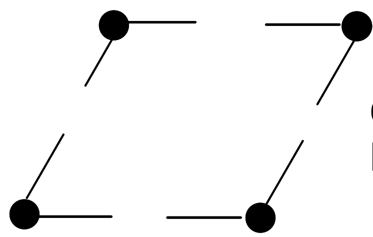


CAN BE DRAWN AS:
This collection of symbols denotes four different types of <u>C_2 axes</u> and the <u>mirrors</u> which are solid lines

SYMBOL is cmm(2)

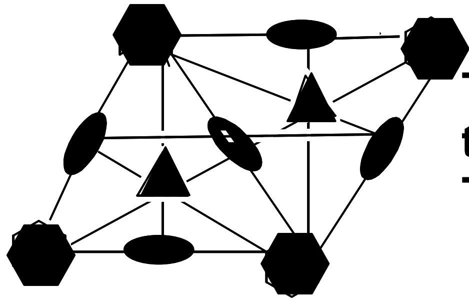
c means centered and two m symbols mean two types of mirror planes (the 2 in parentheses is redundant because the C_2 axes are generated by the mirrors)

5. p6(mm32) - hexagonal 2-D lattice



ONE UNIT CELL OF THE HEXAGONAL LATTICE

CAN BE DRAWN AS



These symbols are for C_6, C_3 and three types of C_2 axes
The lines are mirrors

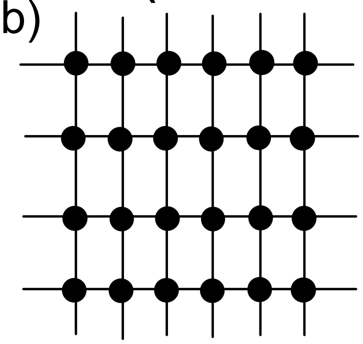
SYMBOL is p6(mm32)

p means primitive and C_6 is the higest order symmetry element - the others can be generated from it

EXAMINE THE 2-D LATTICES WITH CENTERING IN MIND:

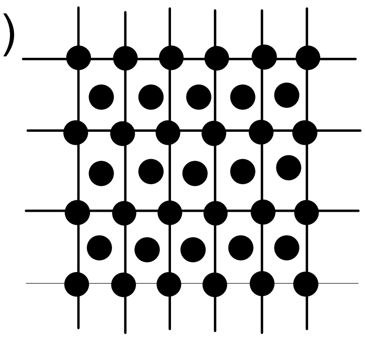
(a and b are not equal but 90 degree angles)

(b)

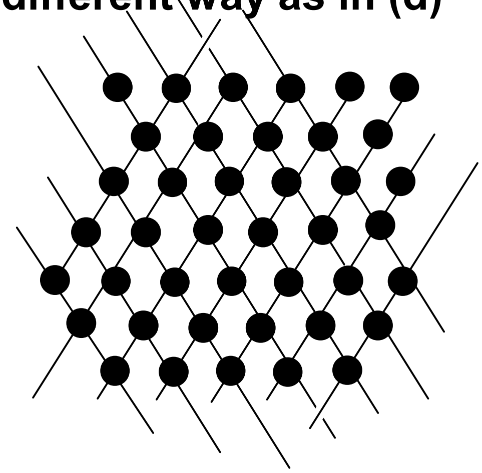


PRIMITIVE
RECTANG.
PLUS
CENTERING
LEADS TO
(e)

(e)



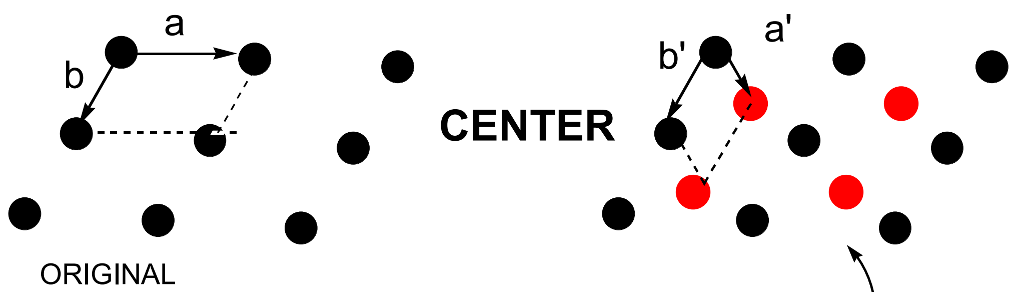
We know that adding the centering point (which is really just a glide) leads to new symmetry because we can draw it a different way as in (d)



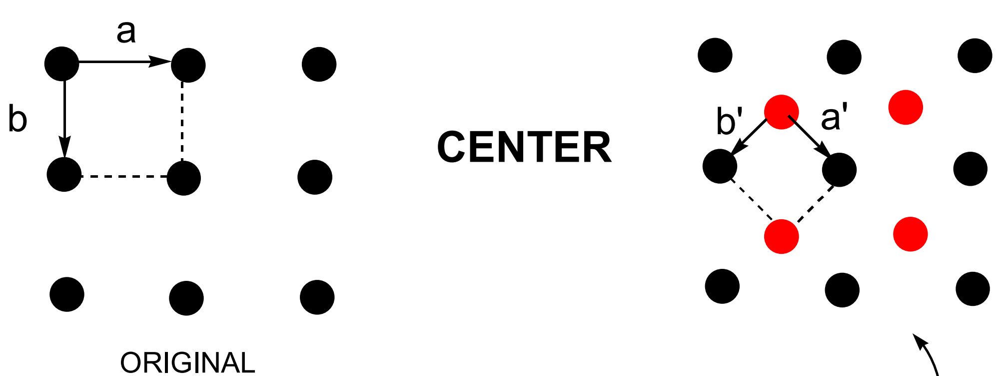
(a and b are equal but not 90 degree angles)

<u>Main Point</u> is that , in this particular case, the Primitve lattice + Centering Generated a New Lattice Type
IS THIS ALWAYS TRUE?

So, with this in mind, go back and center the oblique primitive lattice and the square lattice



THIS IS MERELY A MORE DENSE VERSION OF THE ORIGINAL OBLIQUE UNIT CELL WITH 1/2 OF THE VOLUME
<u>NO NEW SYMMETRY!</u>



MORE DENSE SQUARE LATTICE
1/2 VOLUME OF ORIGINAL
NO NEW SYMMETRY

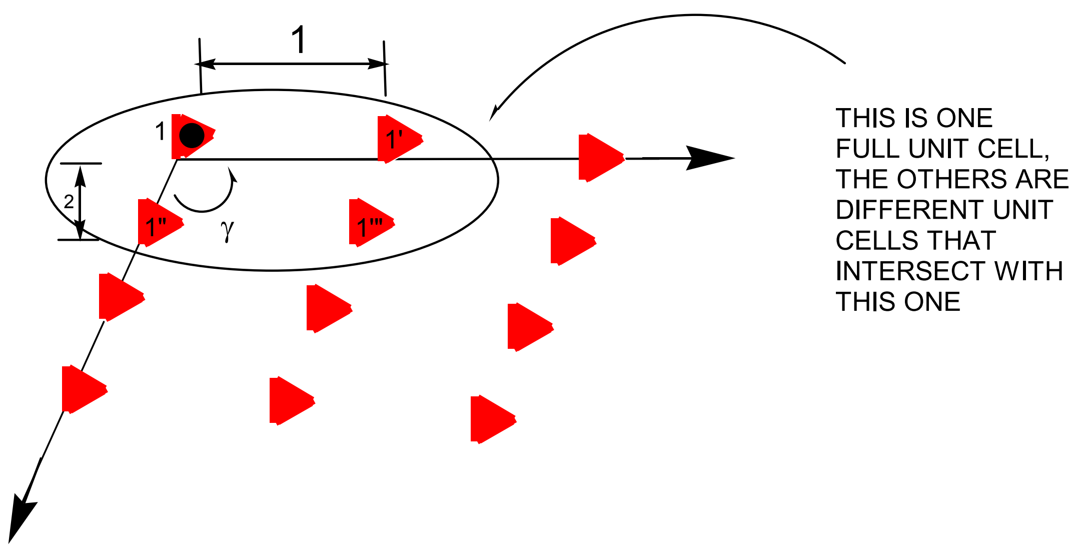
THE FIVE TYPES OF 2-D LATTICES CAN BE USED TO CONSTRUCT <u>SEVENTEEN 2-D SPACE SYMMETRIES (OR SPACE GROUPS) BY ADDING SYMMETRY ELEMENTS TO THE UNIT TRANSLATIONS AND EXISTING UNIT CELL RESTRICTIONS</u>

p1

Unit translations only
No other symmetry

UNIT CELL TRANSLATIONS 1 AND 2 (or a & b) ARE NOT EQUIVALENT AND THE ANGLE γ BETWEEN THEM IS ARBITRARY (IT IS NOT 90)

FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE UNIT TRANSLATIONS

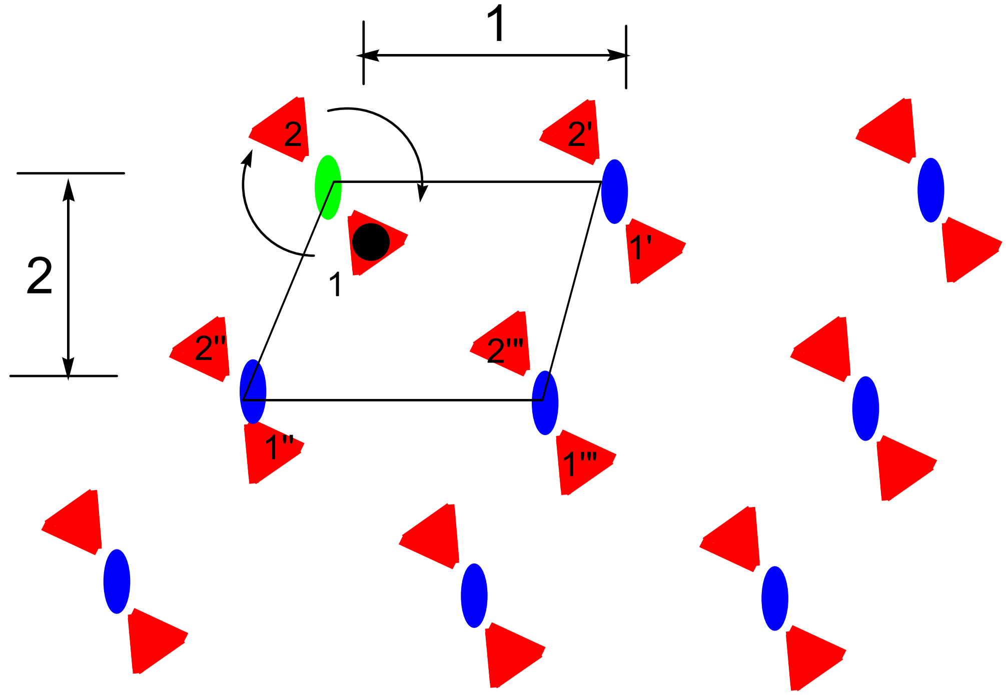


p2

ADD A C_2 TO P1

UNIT CELL TRANSLATIONS 1 AND 2 (or a & b) ARE NOT EQUIVALENT AND THE ANGLE γ BETWEEN THEM IS ARBITRARY <u>BUT WE ADD A C_2 AXIS AT THE CORNERS</u>

1 FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE FIRST C_2 AND APPLY TRANSLATIONS AFTER THAT.

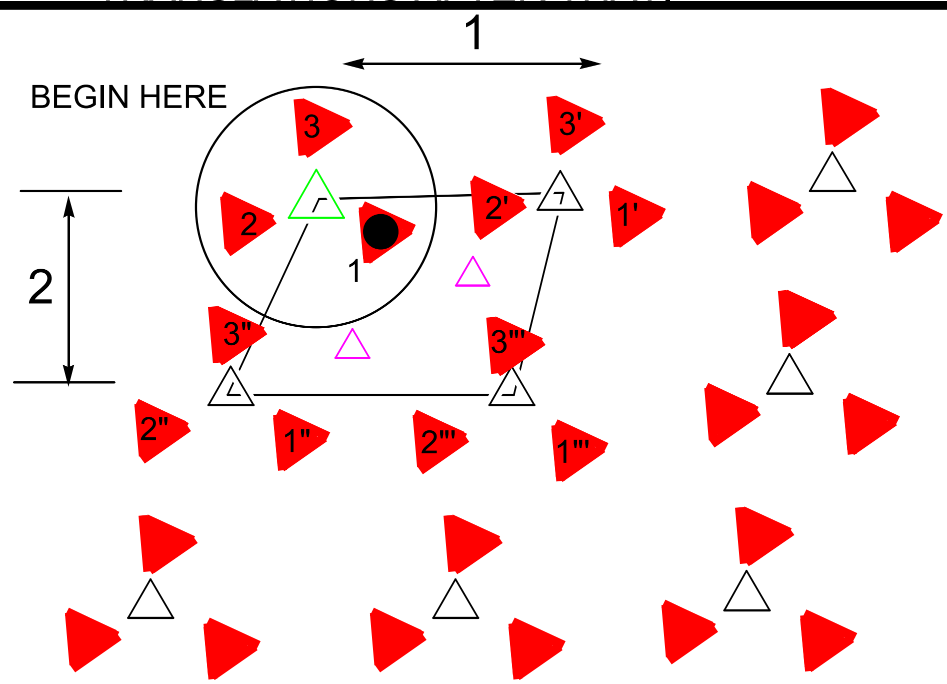


THESE C_2'S ARE GENERATED BY APPLYING THE UNIT TRANSLATIONS TO

p3

ADD A C_3 TO UNIT TRANSLATIONS
a = b and angles are 60° and 120°

1 FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE FIRST C_3 △ AND APPLY TRANSLATIONS AFTER THAT.



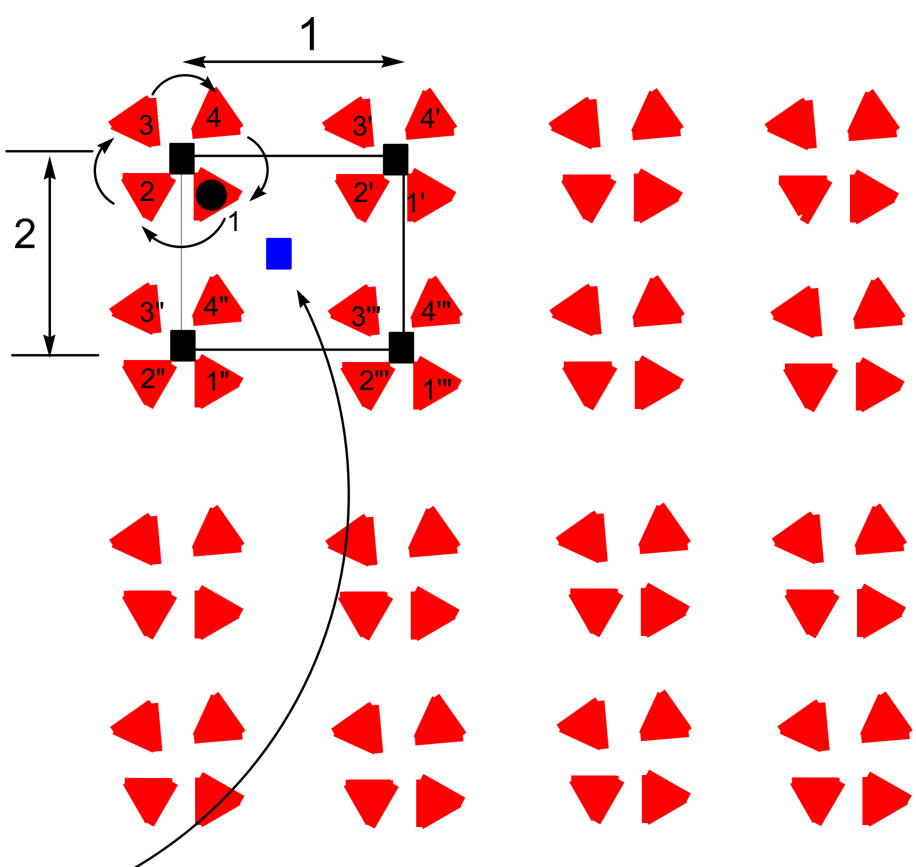
THESE C_3'S △ ARE GENERATED BY APPLYING THE UNIT TRANSLATIONS TO △ AND THESE △ ARE GENERATED BY THE COMBINATION OF UNIT TRANSLATIONS AND APPLICATION OF △

p4

ADD A C_4 TO UNIT TRANSLATIONS
IN THIS CASE 1=2 (OR a = b)

UNIT CELL TRANSLATIONS 1 AND 2 (or a & b) ARE EQUIVALENT AND THE ANGLE γ BETWEEN THEM IS 90° - ADD A C_4 AXIS AT THE CORNERS

FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE FIRST C_4 ■ AND APPLY TRANSLATIONS AFTER THAT.

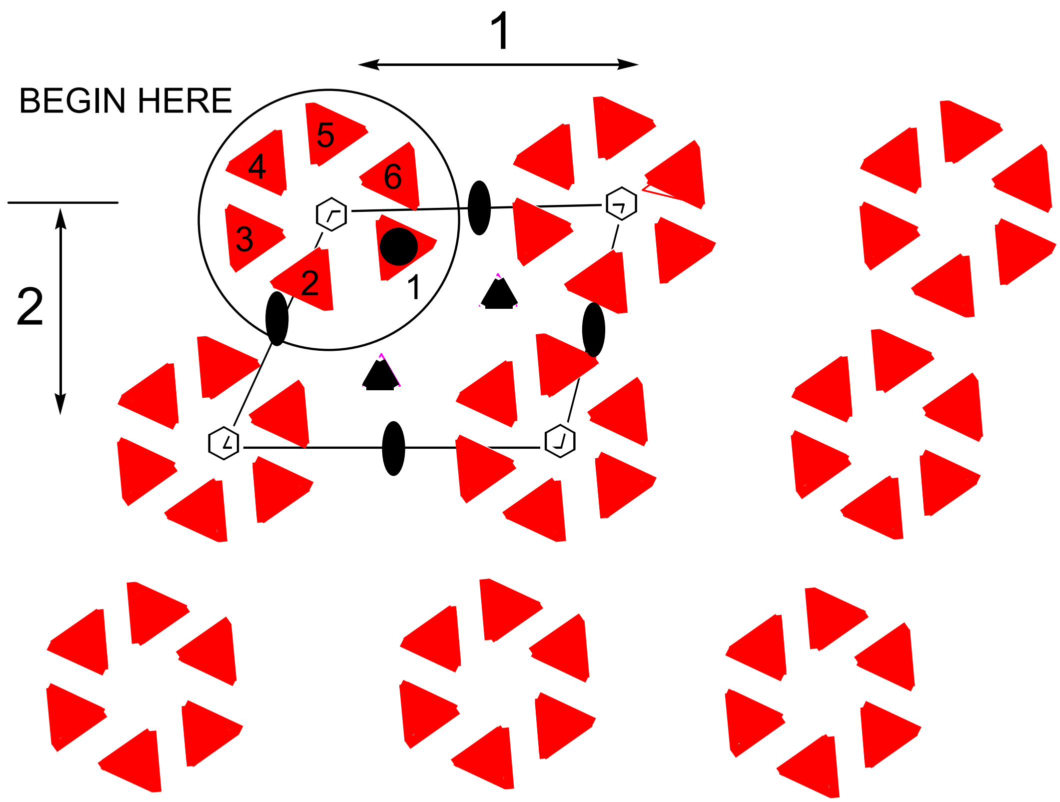


THIS C_4 IS GENERATED BY THE ORIGINAL C_4 PLUS THE UNIT CELL TRANSLATIONS

p6

ADD A C_6 TO UNIT TRANSLATIONS

1 FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE FIRST C_6 ⬡ AND APPLY TRANSLATIONS AFTER THAT.



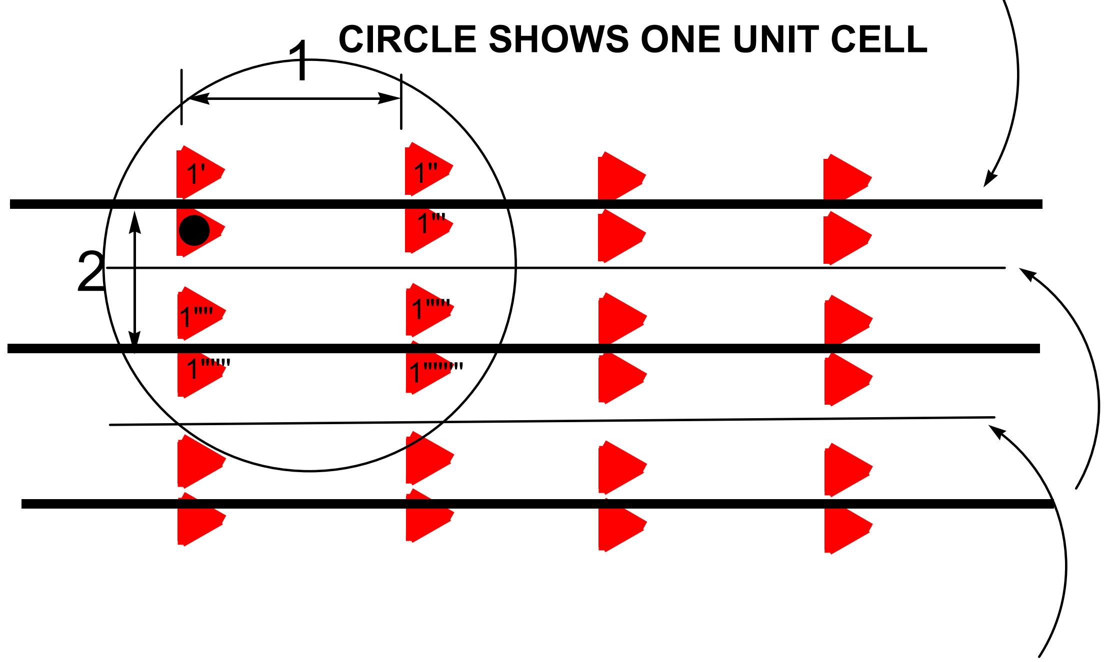
The C_6 AND THE UNIT CELL TRANSLATIONS GENERATE THE C_3's ▲ and the C_2's ⬮

BESIDES ADDING ROTATIONAL AXES, <u>ONE CAN ADD REFLECTIONS</u> (MIRROR LINES IN 2-D) CAN ONLY BE DONE FOR NON-OBLIQUE LATTICES WITH 90°, 60°, AND 120°

pm

Add one parallel mirror line to the unit translations of the rectangular lattice

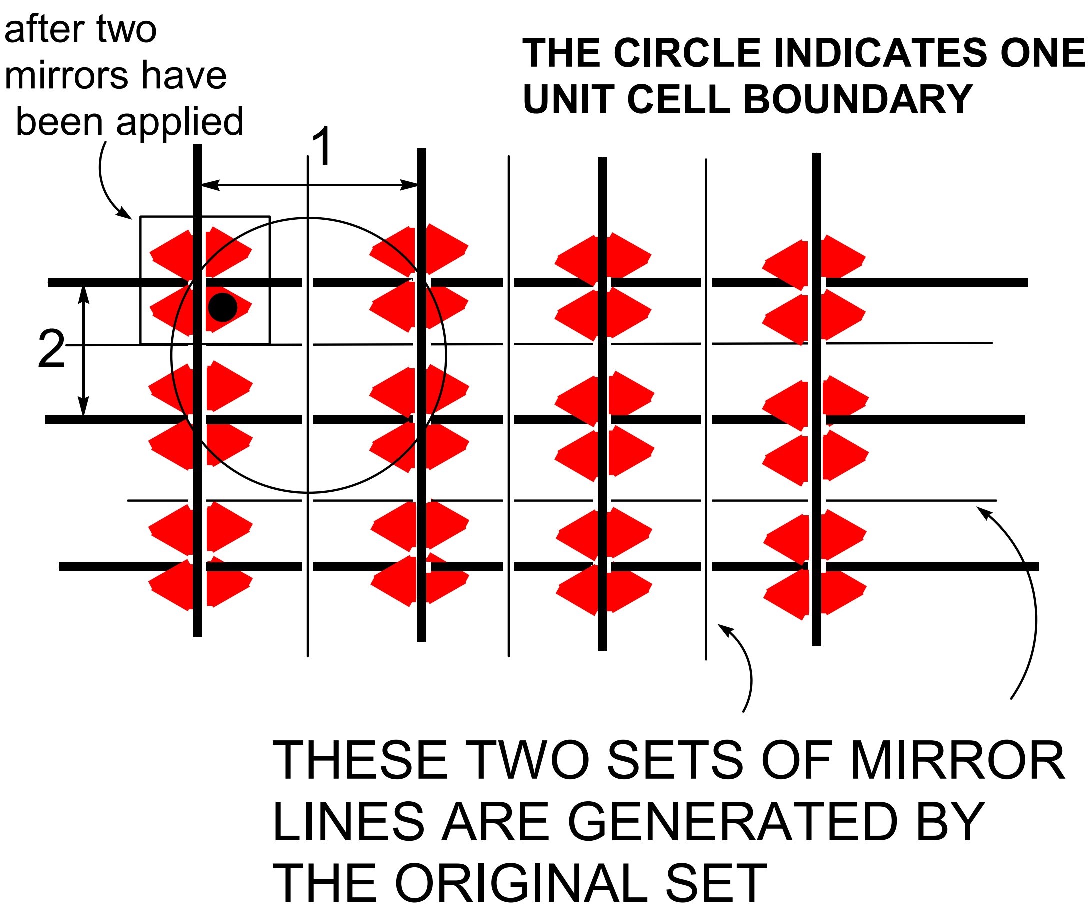
FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE PARALLEL MIRROR LINE AND THE UNIT TRANSLATIONS



THIS SET OF MIRROR LINES IS GENERATED BY THE ORIGINAL SET pmm

Add to the unit translations of the rectangular lattice, <u>one parallel mirror line and a perpendicular mirror line</u>

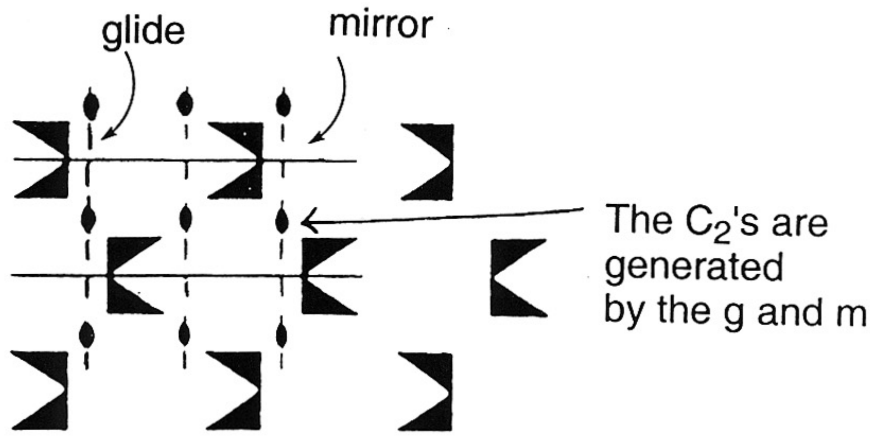
FOLLOW THE MOVEMENT OF THIS ONE WITH APPLICATION OF THE TWO MIRROR LINES (DARK LINES) AND THE UNIT TRANSLATIONS



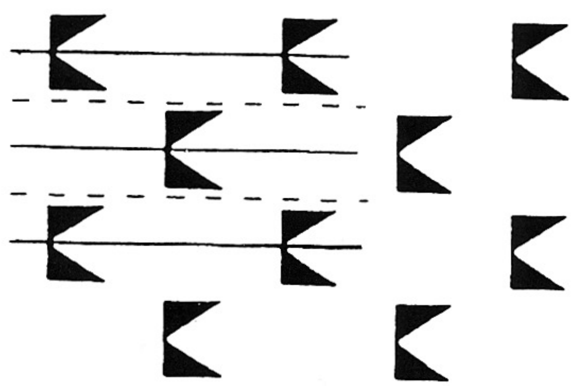
So far, this gives us 9 uniques 2-D Space Symmetries or Space Groups: p1, p2, p3, p4, p6, pm, pmm, pg, pgg

Now we need to add a combination of mirror and glide lines.

pmg



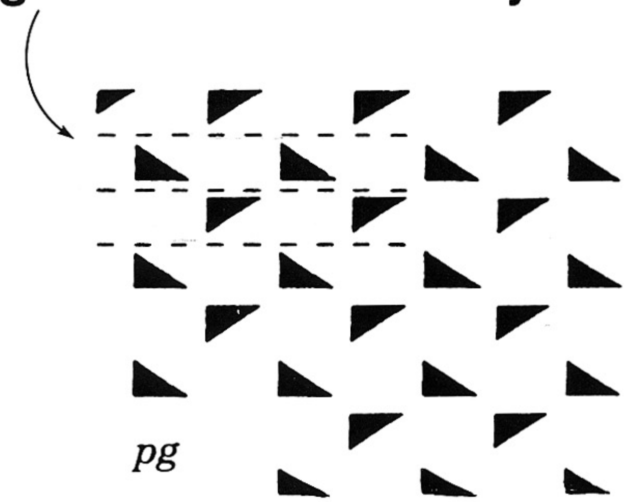
cm



pg

Add to the unit translations of the rectangular lattice, one parallel glide line

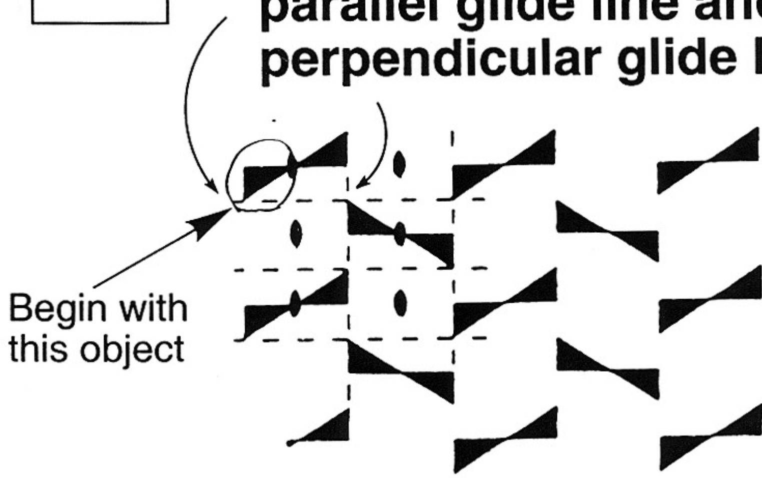
(glides are denoted by a dashed line)



pg

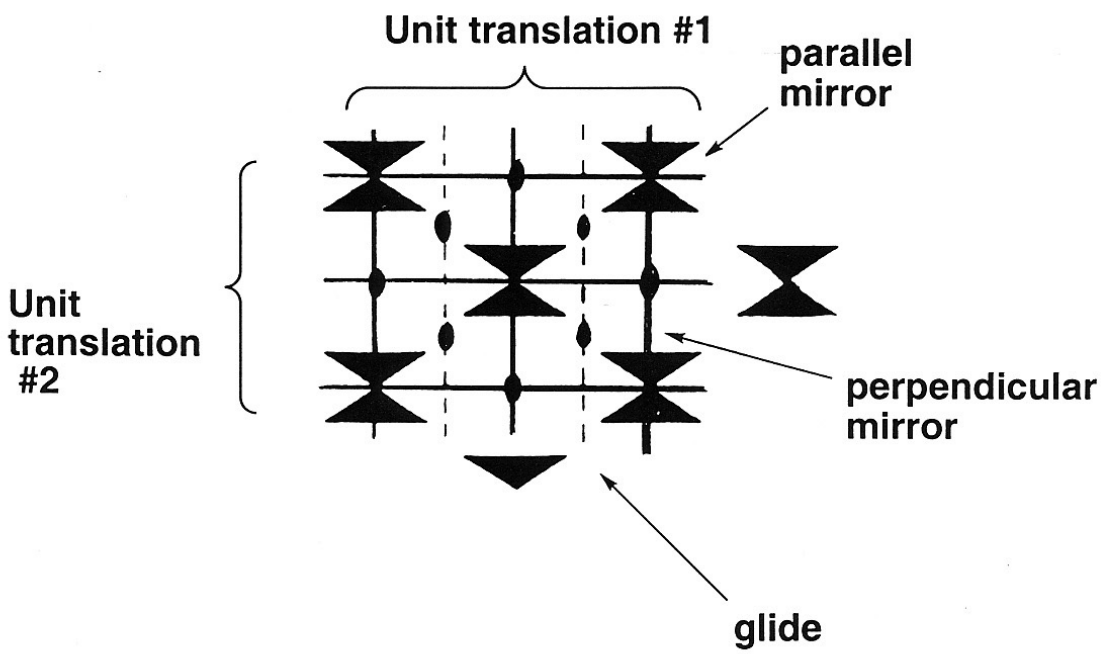
pgg

Add to the unit translations of the rectangular lattice, one parallel glide line and a perpendicular glide line



pgg

cmm



If one places the two perpendicular mirror lines and the glide into the diagram, it leads to a c-centered lattice type. There is no need to specify the glide in the symbol for the space symmetry, because "c" takes care of it.

THERE ARE 5 REMAINING 2-D SPACE SYMMETRIES THAT ARE OBTAINED BY ADDING MIRROR LINES (REFLECTIONS) TO THE p3, p4 and p6 groups.

p3m1	p31m
MIRRORS PASS THROUGH ALL THE 3-FOLD AXES	MIRRORS PASS THROUGH <u>EVERY OTHER</u> 3-FOLD AXIS

THE NUMBER "1" HAS NOTHING TO DO WITH SYMMETRY. IT IS USED TO DISTINGUISH REFLECTIONS THAT PASS THROUGH ALL 3-FOLD AXES FROM THOSE THAT PASS THROUGH ONLY ALTERNATE 3-FOLDS. THIS DIFFERENCE LEADS TO TWO DIFFERENT SPACE SYMMETRIES - THE ONES SHOWN ABOVE.

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p4m	p4g
MIRRORS ARE ALONG UNIT CELL EDGES AND <u>PASS THROUGH ALL THE 4-FOLD AXES</u>	MIRRORS BISECT THE UNIT CELL EDGES AND <u>PASS BETWEEN THE 4-FOLD AXES</u>

THESE TWO ARE DIFFERENT BECAUSE OF WHERE THE MIRRORS ARE PLACED. THE SECOND SYMBOL ACTUALLY DOESN'T EVEN INCLUDE A MIRROR BECAUSE THE GLIDE PERPENDICULAR TO THE C_4 GENERATES THE MIRROR AND IT IS EASIER TO WRITE THE GLIDE THAN TO FIND ANOTHER WAY TO WRITE THE DIAGONAL MIRROR THAT PASSES THROUGH THE C4 AND GENERATES THE GLIDE.

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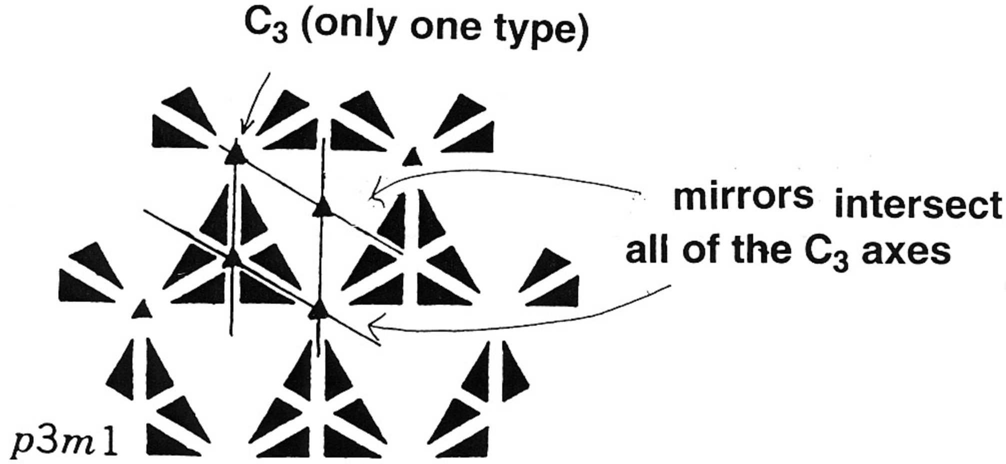
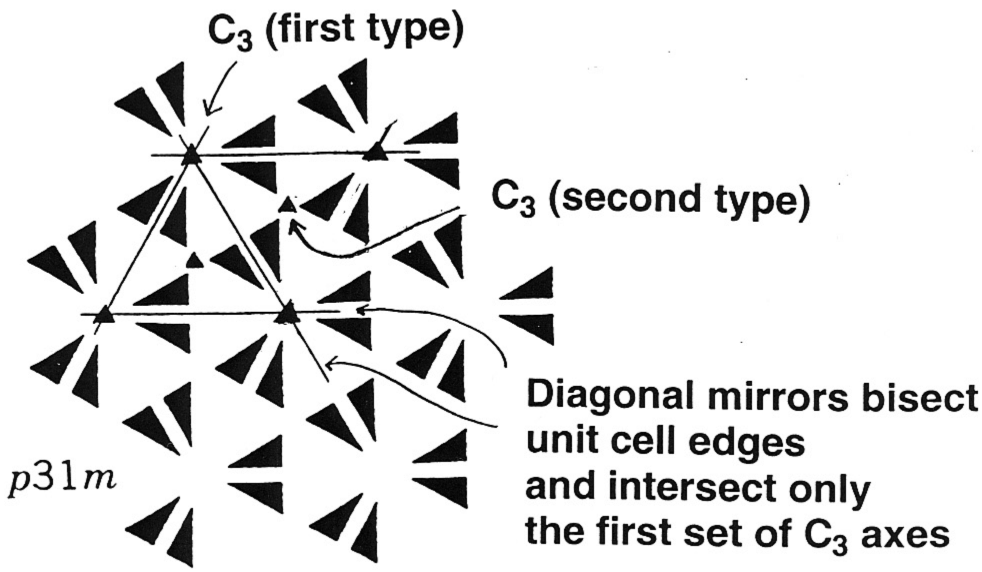
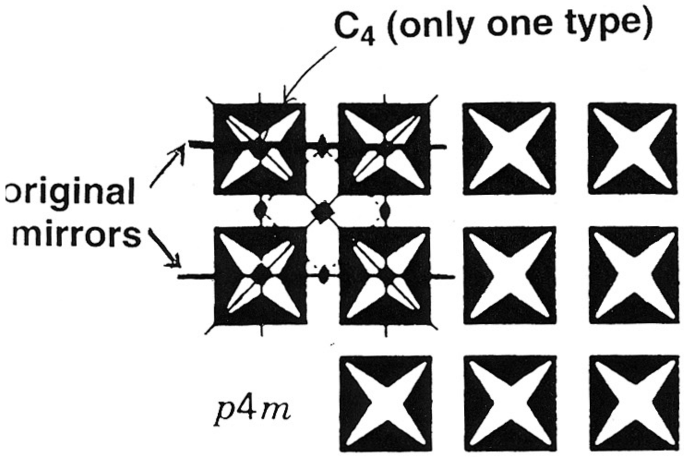
p6m

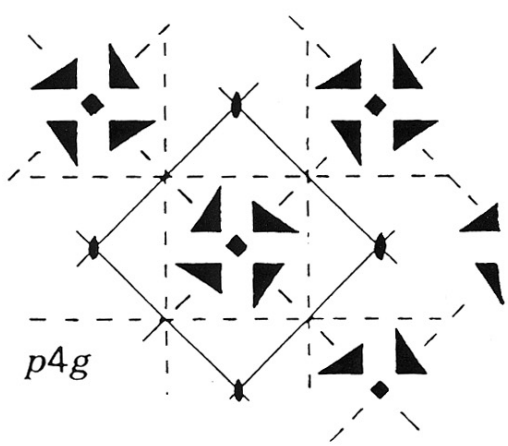
MIRRORS ALONG THE UNIT CELL EDGE PASS THROUGH ALL THE 6-FOLD AXES

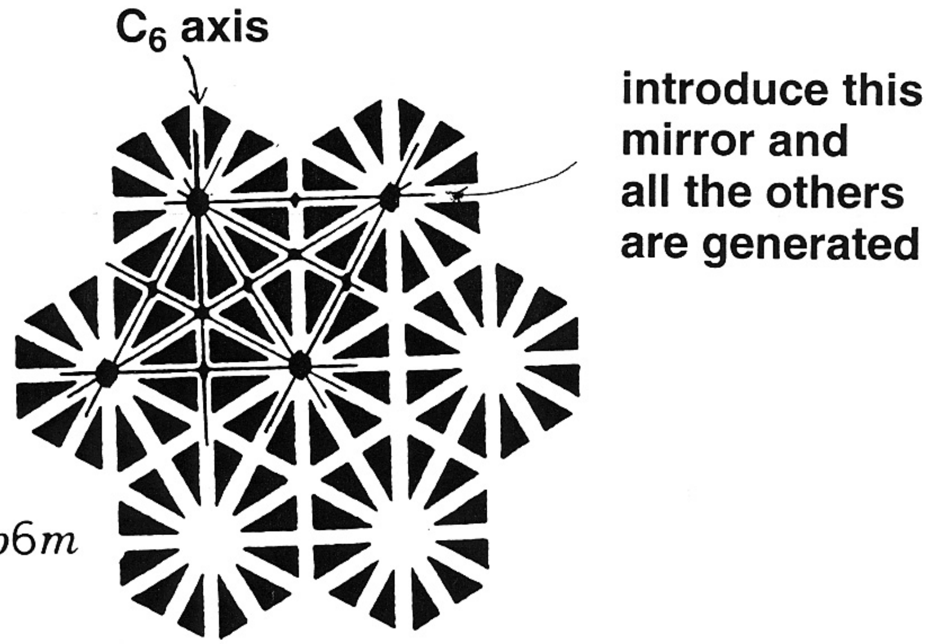

C3 (only one type)
mirrors intersect
all of the C3 axes
p3m1


C3 (first type)
C3 (second type)
Diagonal mirrors bisect
unit cell edges
and intersect only
the first set of C3 axes
p31m

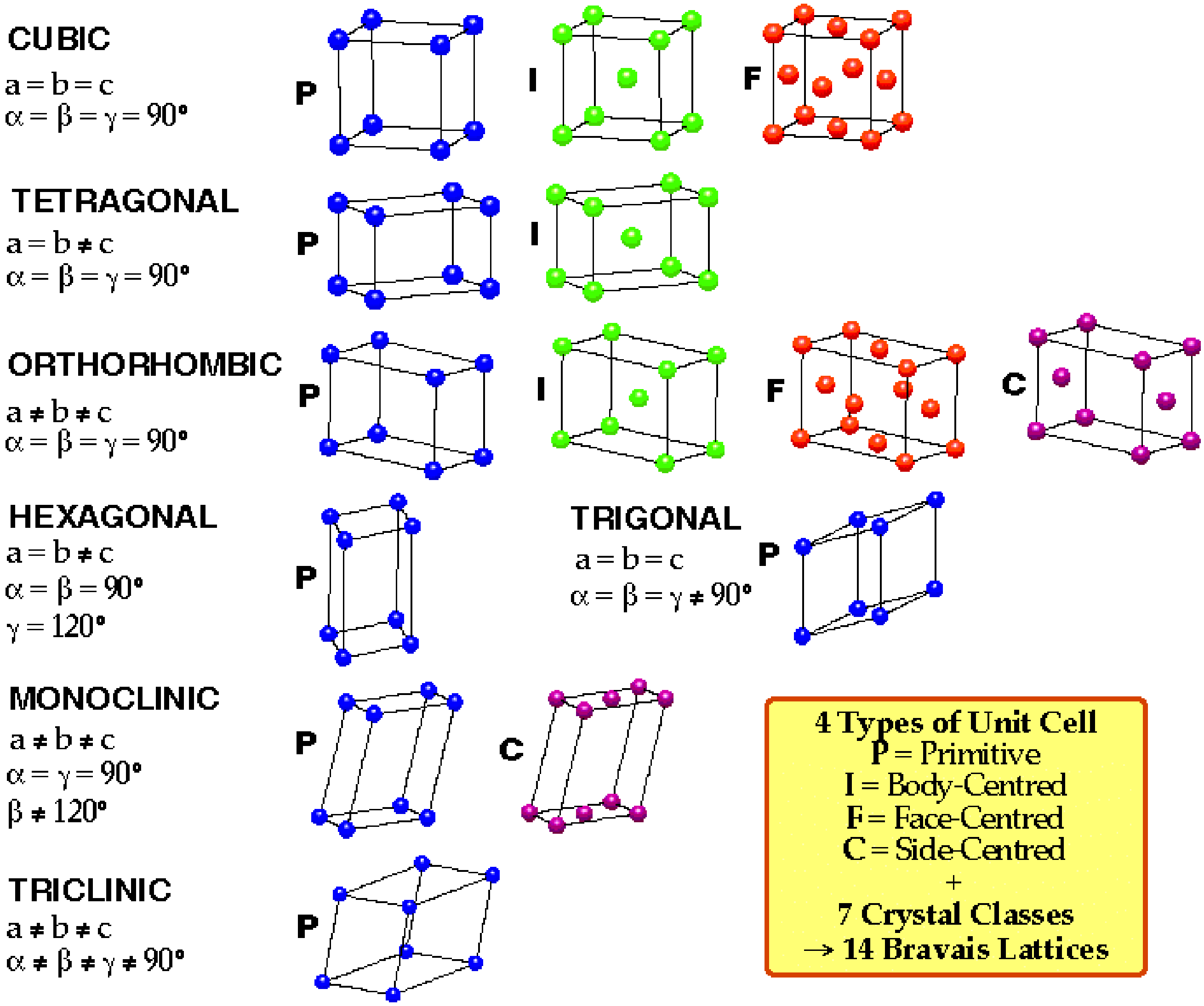


C4 in the middle and C2's along the edges and diagonal are generated by other symmetry




C_6 axis
introduce this
mirror and
all the others
are generated

$p6m$


CUBIC
a = b = c
α = β = γ = 90°
P
I
F
TETRAGONAL
a = b ≠ c
α = β = γ = 90°
P
I
ORTHORHOMBIC
a ≠ b ≠ c
α = β = γ = 90°
P
I
F
C
HEXAGONAL
a = b ≠ c
α = β = 90°
γ = 120°
P
TRIGONAL
a = b = c
α = β = γ ≠ 90°
P
MONOCLINIC
a ≠ b ≠ c
α = γ = 90°
β ≠ 120°
P
C
TRICLINIC
a ≠ b ≠ c
α ≠ β ≠ γ ≠ 90°
P
4 Types of Unit Cell
P = Primitive
I = Body-Centred
F = Face-Centred
C = Side-Centred
+
7 Crystal Classes
→ 14 Bravais Lattices